The Concept of “Essential” General Validity in Wittgenstein’s *Tractatus*

Brice Halimi

Paris West University (IREPH) & SPHERE

Abstract

In the *Tractatus*, Wittgenstein describes the general validity of logical truths as being “essential,” as opposed to merely “accidental” general truths. He does not say much more, and little have been said about it by commentators. How to make sense of the essential general validity by which Wittgenstein characterizes logic? This paper aims to elucidate this crucial concept.

Keywords: Wittgenstein; *Tractatus*; essential general validity; accidental general validity; series of forms; form-series generality; logical necessity; tautologies.

1 Logical generality and logical validity

This paper is a comment on the following passage of the *Tractatus*:

6.1231. The mark of a logical proposition is *not* general validity.

To be general means no more than to be accidentally valid for all things. An ungeneralized proposition can be tautological just as well as a generalized one.

6.1232. The general validity of logic might be called essential, in contrast with the accidental general validity of such propositions as ‘All men are mortal’. Propositions like Russell’s ‘axiom of reducibility’ are not logical propositions, and this explains our feeling that, even if they were true, their truth could only be the result of a fortunate accident.¹

¹[Wittgenstein, 1974], p. 63.
The text above puts forward a new characterization of the “general validity of logic,” which implies a new characterization of both logical generality and logical validity. The Tractarian concept of essential general validity is not clear by itself. Few commentators, however, have dwelled upon its exact content. Anscombe does not analyze it. Robert Fogelin, in his chapter devoted to necessity in the *Tractatus*, focuses on 6.37 (“The only necessity that exists is *logical* necessity”) and does not explain the peculiar concept of logical validity to which Wittgenstein alludes. Michael Morris skims past the issue. Peter Hacker mentions “the *essential* validity of logic,” but only as a rebuttal of Russell, and without explaining its positive content apart from referring 6.1231 back to 6.113. Nothing can be found specifically about 6.1231-6.1232 in [Landini, 2007], [Potter, 2009] or [Kuusela and McGinn, 2011]. The doctrine of showing about necessity (namely the doctrine, expressed at 6.12, that tautologies show the “formal” properties of language and the world) upstages the issue of the actual meaning of logical validity. However, the doctrine of showing—together with the Hauptsatz (5.4) that there are no such things as “logical constants”—is but one side of a two-pronged reflection. The other, more positive side lies precisely at 6.1231. An indication of that, as noticed by Anscombe, is the fact that Wittgenstein mentions certainty instead of necessity at 5.525. Logical necessity is not to be dispensed with. However, Anscombe does not say much more. The purpose of this paper is to analyze Wittgenstein’s characterization of logical generality and to show how the Tractarian concept of logical validity distances itself from Russell’s in a positive way.

There are at least three reasons why the Tractarian concept of essential general validity has been relatively sparsely discussed. Firstly, as just mentioned, the doctrine of showing, and the mainly negative philosophy of logic advocated in the *Tractatus*, have been the main focus of attention. Secondly, Russell’s theory of types has seemed to be the main target of Wittgenstein’s criticism in the text being considered. Finally, and most importantly, many commentators have considered that 6.1231-6.1232 is really about logical *necessity*—about the kind of necessity peculiar to logical truths as tautologies:

Logical propositions need not be general, since a tautology containing names—an ‘ungeneralized proposition’ (6.1231c)—may be valid in virtue of its form. And when logical propositions are general, they are not ‘accidentally’ valid, do not *happen* to be true of everything, but have ‘essential validity’, because they treat of the ‘formal’ aspects

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4See [Hacker, 1989], pp. 38 and p. 47.
5[Anscombe, 1959], p. 158.
6As an example, see [Stenius, 1960], p. 201.
of the world (6.12a).\footnote{[Black, 1964], p. 326.}

All three above-mentioned reasons are often associated: Wittgenstein would attack the theory of types in \textit{Principia Mathematica}, because the latter epitomizes the logical perspective that tries to express the formal features of the world as general facts through propositions which, although generalized, cannot be but contingently true, if true at all. Putting forward essential general validity and setting it apart from accidental generality would amount to criticizing the endeavor to describe as a fact, however general, what can only be shown as part of the scaffolding of the world.

As known, the opposition between essential and accidental general validity of logical propositions is elaborated in the \textit{Tractatus} in overt polemic with Russell’s conception of logic. In effect, the above mentioned inadequacy of the logicist reduction of arithmetic follows as a simple corollary from what Wittgenstein considered one of the main flaws of Russell’s theory of types: among the axioms of the theory, there are propositions that, despite their complete generality, do not satisfy the basic requirement of logical validity, i.e. truth in all possible worlds. [...] According to Wittgenstein, an essential general validity is one that does not depend whatsoever on the real configuration of the world, on the particular arrangement of objects which actually occurs.\footnote{[Frascolla, 1994], pp. 38-39.}

This reading is followed by Roger White.\footnote{“What is peculiar about a truth of logic is that you can tell that it is true from the symbol alone, so that the proposition is true regardless of the way the world is, i.e., necessarily true.” ([White, 2006], p. 106)} It is mistaken, however. First, 6.12 does not say that tautologies depict or even show the formal aspects of the world, but that certain propositions being tautologies shows the formal properties of the world. So the tautological nature of certain propositions cannot be explained in terms of formal properties, let alone necessary features of the world: It is the other way around.\footnote{Cora Diamond (“Throwing Away the Ladder: How to Read the \textit{Tractatus}”, chapter 6 of [Diamond, 1991]) rightly insists against Hacker that logical necessity has nothing to do with “features of reality” whatsoever, but pertains to linguistic \textit{constructions}. She writes, about 6.37 and 6.375: “Logical necessity is that of tautologies. It is not that they are true because their truth conditions are met in all possible worlds, but because they have none. ‘True in all possible worlds’ does not describe one special case of truth conditions being met but specifies the logical character of certain sentence-like constructions formulable from sentences.” ([Diamond, 1991], p. 198) However, she adds: “But the remark that there is only logical necessity is itself ironically self-
to pinpoint first and foremost, before a certain kind of necessity, a certain kind of generality. When Wittgenstein says (at 6.1231) that being tautological does not imply being generalized, he only means that a tautology does not have to be universally quantified, and actually that the generality of a tautology has nothing to do with quantification. This is borne out by the examples given by 6.1232, and rightly acknowledged by Marie McGinn:

One of the main themes of Wittgenstein’s reflections on the propositions of logic in the *Notebooks* is the attempt to make clear the distinction between the propositions of logic and fully generalized, material propositions in which all the constants have been replaced by variables. Clarification of this distinction is fundamental to Wittgenstein’s overall aim to make clear that the sort of generality that belongs to the propositions of logic is quite distinct from the merely accidental generality of general empirical propositions. The generality that characterizes logic has nothing to do with general truth, but with the generality of logical form, that is, with something that abstracts from all content.\(^\text{11}\)

Admittedly, Wittgenstein’s target is partly Russell’s proposal simply to replace necessity with universal validity, notably in the paper “Necessity and Possibility,” drawn from a talk that Russell gave at the Oxford Philosophical Society in October 1905:

It is possible to regard a proposition as necessary when it is an instance of a type of propositions all of which are true. For example, “Socrates is either a man or not a man” may be called necessary on the ground that the statement remains true if we substitute anything else in place of Socrates. […] To make our definitions of necessity and possibility precise, in this theory, it is natural to regard necessity and possibility as not attaching to propositions, but to propositional functions, that is, to propositions with an indeterminate subject. Thus we may define as follows:

“The propositional function ‘\(x\) has the property \(\phi\)’ is necessary if it holds of everything; it is necessary throughout the class \(u\) if it holds

destructive. […] In so far as we grasp what Wittgenstein aims at, we see that the sentence-form he uses comes apart from his philosophical aim. If he succeeds, we shall not imagine necessities as states of affairs at all. We throw away the sentences about necessity; they really are, at the end, entirely empty.” (ibid.) If one were to follow Diamond’s reading, one should conclude, not only that logical necessity is not descriptive, but that it is undescribable. Yet it entirely remains to understand what the necessity of tautologies consists in exactly.

\(^{11}\)[McGinn, 2006], p. 59.
Thus, Russellian general validity is always the general validity of some propositional function \( \varphi \), which itself amounts to the truth of the corresponding universally quantified proposition \((x) \varphi x\).\(^{13}\)

However, Russell-averse as it may be, 6.1231-6.1232 does not seek to vindicate necessity against Russell’s rendering of it. Indeed, the essential validity put forward in the Tractatus remains an essential general validity. Wittgenstein’s purpose at 6.1231-6.1232 is not to rebuke Russellian logical generality in favor of logical necessity understood as tautologyhood. On the contrary, it is to bring out the right notion of logical generality against the quantificational generality on which Russell relies exclusively.

There is a traditional assumption that generality and necessity can both be equally considered as criteria of logical validity, and are, as such, equivalent. In particular, Kant writes in his Logic:

> If [...] we put aside all cognition that we have to borrow from objects and merely reflect on the use just of the understanding, we discover those of its rules which are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. [...] And from this it follows at the same time that the universal and necessary rules of thought in general can concern merely its form and not in any way its matter. Accordingly, the science that contains these universal and necessary rules is merely a science of the form of our cognition through the understanding, or of thought. And thus we can form for ourselves an idea of the possibility of such a science, just as we can of a universal grammar, which contains nothing more than the mere form of language in general, without words, which belong to the matter of language.

Now this science of the necessary laws of the understanding and of reason in general, or what is one and the same, of the mere form of thought as such, we call logic.\(^{14}\)


\(^{13}\)Gregory Landini (see [Landini, 2007], Appendix B) holds that Russell defines as necessary any true fully generalized proposition in the sense of its first-order and second-order universal generalization. For instance, \( \varphi x \supset \psi x . \varphi a ; \supset . \psi a \) (1R) is Russell-necessarily true because its full universal generalization \( (\varphi)(\psi)(y): . \varphi x . \psi x ; \supset . \psi y \) (2R) is true. One possible objection is that necessity then becomes the feature of a sentence, i.e., of the particular linguistic expression of a proposition. For suppose \( Px \) is \( \varphi x \supset \psi x \). Then \( (1_P) \) is equivalent with \( (x)Px . \varphi a ; \supset . \psi a \) (1_P), whereas \( (2_P) \), which is true, is not equivalent with the full universal generalization of \( (1_P) \), \( (P)(\varphi)(\psi)(y): (x)Px . \varphi y ; \supset . \psi y \) (2_P), which is false.

As one can see, Kant indiscriminately describes logic by its formal generality and by its absolute necessity. Kant and Russell obviously do not conceive of the generality of logic in the same way. Kant sees logic as a canon (a body of rules), whereas Russell sees it as a science (a system of truths). As a consequence, the generality of logic, to Kant, lies in the applicability of logical rules independently of the content of the judgments at stake. In contrast, to Russell, it lies in the unrestrictedness of logical truths as pertaining to all entities whatsoever. To Kant, logic is contentless, whereas to Russell it has a content with the widest scope. As is well known, Kant distinguishes between transcendental logic and formal logic, and the validity of the transcendental principles of pure understanding cannot be established for itself, “but always only indirectly through relation of these concepts to something altogether contingent, namely, possible experience.”\(^{15}\) However, transcendental logic is no less general than formal logic, because the domain ruled by formal logic cannot be seen as extending the domain ruled by transcendental logic. No domain properly speaking can embrace and enlarge the totality of possible experience. In that setting, the generality of formal logic has to be reckoned through something other than its universality, namely: Formal logic states conditions of possibility of thought as such. To Kant, necessity is a way to account for formal generality from within a non-formal conception of logic. Russell’s strategy goes the other way round: The generality of logic allows Russell to express the special validity of logical truths from within an anti-psychologistic conception of logic, i.e., despite “the fact that the division of judgments into necessary, assertorical, and problematic is, in the main, based upon error and confusion.”\(^{16}\) Unrestricted generality is Russell’s way of reckoning the existence of logical truths without committing himself to some kind of necessity that could not be captured formally.\(^{17}\)

In 6.1231-6.1232, Wittgenstein certainly follows the philosophical tradition, starting with Kant, which associates the formal generality of logic with its particular validity. The *Tractatus* does not aim to depart from this tradition. It rather deepens it so as to connect it back to its genuine root and to clarify the combination of generality and necessity peculiar to logical truths. Explaining this combination will be the main concern of the rest of this paper.

\(^{15}\) [Kant, 1929], p. 592 (A737/B765).

\(^{16}\) [Russell, 1992a], p. 508.

\(^{17}\) About Russell’s wariness about modal notions, which does not amount to “wholesale eliminativism,” see [Shieh, 2011], pp. 3-4.
2 Essential generality

“Essential” general validity has first to do with generality. On that score, the canonical text in the *Tractatus* is 5.501, which distinguishes three ways of determining the values of a propositional variable:

We can distinguish three kinds of description: 1. direct enumeration, in which case we can simply substitute for the variable the constants that are its values; 2. giving a function $f(x)$ whose values for all values of $x$ are the propositions to be described; 3. giving a formal law that governs the construction of the propositions, in which case the bracketed expression has as its members all the terms of a series of forms.$^{18}$

A series of forms (4.1252) is a series of terms (or propositions) that is ordered by “internal relations” shown in the symbols for the terms, so that each term (except the first) can be deduced from the previous one according to formal features expressing the same formal law. The example given at 4.1273 is: $aRb, (\exists x) aRx.xRb, (\exists x,y) aRx.xRy.yRb, \ldots$; The general term of a series of forms is written: $[a, x, O'x]$ (5.2522).

Given a propositional function $\xi$, Wittgenstein writes $\bar{\xi}$ for the class of all its values. This shift is in no way automatic. What Wittgenstein calls a variable is precisely the determination of a class of propositions as a source of generality. The generality lies entirely in the mode of determination of such a class that has to be described and thus unfolded before it can be ranged.

Wittgenstein apparently considers three main ways to present a domain of generality. The trichotomy is superficial, however. Indeed, direct enumeration is not a mode of determination among others, but the ideal original datum in comparison to which any mode of determination can appear as a particular mode, a particular way of producing a class of objects. It becomes meaningless, however, in the infinite case, which is the main stake of logical generality. The use of a propositional function is the second mode of determination that Wittgenstein reckons with. However, the only determination that such a device (the sign of a function) is able to carry out is purely nominal—unless a magical “range of significance” is supposed to be attached to any function, and given along with it: This is actually Russell’s last resort in “Mathematical logic as based on the theory of types.”$^{19}$ The functional notation of the form $f(x)$ used by Frege and Russell is also criticized at 5.521:

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$^{18}$[Wittgenstein, 1974], p. 49.

$^{19}$“[...] every propositional function has a certain *range of significance*, within which lie the arguments for which the function has values.” ([Russell, 1956], pp. 72-73)
5.521. I dissociate the concept all from truth-functions.

Frege and Russell introduced generality in association with logical product or logical sum. This made it difficult to understand the propositions ‘( $\exists x) fx$ ’ and ‘( $x) fx$ ’, in which both ideas are embedded.\(^{20}\)

The use of functional symbols completely glosses over the task of generating and specifying the class of all its admissible arguments. This task is generally discharged in mathematics (in mathematical analysis) by the theoretic framework in which the “domain of definition” of each function is amenable to complete determination. Mathematical analysis is but a particular case, however, and nothing guarantees such a determination in the general case, i.e., in the context of logic.\(^{21}\)

The sole genuine mode of determination of the values of a variable consists in a series of forms. In that case, the class of all values is properly generated, following the operation “that produces the next term out of the proposition that precedes it” (4.1273). The general term of a series of forms is a variable, and any genuine variable is given through a series of forms. Quantificational generality (( $x) fx$ ) relies on a function notation ( $fx$ , or $f(x)$ ) that does not explain by itself, within its own symbol, how the values of $x$ can be determined, and thus how it makes sense.

The function $f \hat{z}$ can actually be rephrased as the following degenerate series of forms:\(^{22}\)

$$[(fa, z), (fx, y), (f[y/x], z)].$$

This rephrasing allows one to see what is right and what is wrong in unspecified quantification. Indeed, the difference between

$$aRb, \exists x (aRx . xRb), \exists (x, y) (aRx . xRy . yRb), \ldots \quad (1)$$

and

$$\varphi(a), (\varphi(a), \varphi(x)), (\varphi(a), \varphi(x), \varphi(y)), \ldots \quad (2)$$

is obvious. In (1), one has to design ever new variables, so there is no reason why this should be refused in the case of (2), and this is enough to bestow a minimal meaning on the rephrasing above of $f \hat{z}$ as a degenerate series of forms. However, ‘$x$’ and ‘$y$’ have the status of variable in (1), which becomes completely unclear in (2), so that it is impossible to know how to continue. In other words, the variable involved in a propositional function written $fx$ or $f \hat{z}$ (and taken as such) is only the sign of a variable, not a true variable symbol.

In Wittgenstein’s view, the main and probably only logically correct expression of generality resides in the iterability of the operation that underpins a series

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\(^{21}\) It should be noted that the phrase “propositional function” occurs only once in the whole Tractatus, at 5.521.

of forms. The logically correct variable symbols are the variables whose values are specified by a formal law—“form-series variables,” as Thomas Ricketts calls them.\footnote{Ricketts, 2013, p. 136.} The logically correct generality is the generality of the general term of a series of forms, which we shall call \textit{form-series generality}. This is confirmed by Wittgenstein’s \textit{Notebooks}\footnote{Wittgenstein, 1961, 13.10.14, p. 11.} and by his later remarks on the difference between totalities and systems.\footnote{Wittgenstein, 1967, pp. 216-217.} Form-series generality is essential precisely to the extent that it is governed “by an internal relation” (4.1252) and resides “in the symbol itself” of the series (5.525—the connection with 6.113 being obvious). The exact opposite is the “\textit{accidental} generality” carried by the theory of classes (6.031), and to which corresponds the domain of values that quantification merely presupposes. Quantificational generality leaves the propositional variable $\xi$ (the class of all its values) up in the air. In contrast, form-series generality pertains by definition to a form-series variable whose class of values is produced through an operational rule built into the symbol of a series of forms.

Of course, quantificational generality is not dismissed at all by the \textit{Tractatus}.\footnote{See, in particular, 4.0411, 5.1311, and 5.52.} As a notational system, quantification is irreproachable: This is what is to be gathered from 4.0411. The point is elsewhere, in the way in which a notation can be filled in with an actual meaning. In this respect, quantification is certainly a kind of mixed form straddling ordinary language and mathematical language. Quantification, applied to propositional functions, is a very versatile tool which allows one to formalize both “There are two objects which are laid on my desk” and “There are two objects which are a solution of the equation $x^2 = 1$.”\footnote{See [Wittgenstein, 2001], pp. 125-127 and pp. 171-175. On this point, see [Sackur, 2005], pp. 133-141.} As such, the quantificational notation is a powerful tool to uniformize the various aspects of generality in language. As a counterpart of this virtue, quantificational variables cannot by themselves be more than a notational device: They do not show their range by themselves. This is not a problem in the context of ordinary language, or if the framework of the theory of classes is endorsed. However, this cannot do as an exact logical representation of how \textit{logical} generality actually works; This cannot do as an exact analysis of the generality of logical propositions. In the case of an empirical propositional function, it is part of the meaning of the latter that its extension is determined factually. However, in the case of a logical or a mathematical concept, it cannot be left to the world to determine its extension, i.e., to produce the range of values of the corresponding variable.\footnote{Wittgenstein, 1961, 13.10.14, p. 11: “But let us remember that it is the \textit{variables} and \textit{not} the sign of generality that are characteristic of logic.”}
The objection of “accidental generality” that Wittgenstein raises at 6.031, against the logicist reconstruction of numbers as equinumerosity classes of extensions, can be better viewed in that light. Wittgenstein’s criticism aims at two targets here: the use of an abstract extensional generality on the one hand, and on the other the resort to propositional functions having as by chance the right logical properties. Let us indeed consider the definition of $0$. In Frege’s and Russell’s view, non-self-identity evidently secures in a logical way the existence of an empty class, and thus the possibility of defining $0$. In other words, it is self-evident to both Frege and Russell that the following sentence (s), $(x), x = x$, constitutes a logical truth. According to Wittgenstein, it is not, as no concept is able to determine in a logical way the cardinality of its extension. One can conceive of a possible world where each propositional function would be satisfied by at least one object. So, should the variable sign ‘$x$’ be a logical term, (s) would be a logical sentence (because all its constituents would correspond to logical notions) and a true sentence, and yet would not be a logically true sentence. This has less to do with the rejection of the identity sign (5.53-5.534) than with a general discussion of logicality, as proved by 5.5352:

$$\begin{align*}
5.5352 & \text{ In the same way people have wanted to express, ‘There are no things’, by writing ‘} \sim(\exists x), x = x.\text{ But even if this were a proposition, would it not be equally true if in fact ‘there were things’ but they were not identical with themselves?29} \\
& \text{Thus, Wittgenstein implicitly confronts logicists with the perspective that the truth of a logical sentence does not imply the logical truth of that sentence. The conclusion to draw is that (s) is not really a logical sentence, because a variable sign such as ‘$x$’ is not a logical term as such.30 Everything hinges on the way a variable is introduced and used: on the existence of rules for its referentiality, i.e., on the corresponding “mode of signification” (in the sense of 3.322). Only a series of forms can specify in a principled way the range of values of a variable. Only form-series variables uncover what enables a proposition to be general: This is exactly why they can be described as an “essential” mode of signification, i.e., as a mode of signification based on the “essential features” of generality, in the sense of 3.34-3.341: “Essential features [of the propositional sign] are those without which the proposition could not express its sense.”31 Essential generality can be defined as the generality of a proposition expressed by essential features of the symbol of generality. In contrast, quantificational generality is a blank check that expresses its purported sense only superficially and thus qualifies as accidental} \\
\end{align*}$$

29[Wittgenstein, 1974], p. 53.
30This is actually, in a way, Tarski’s conclusion: The interpretation of a variable changes as one shifts from one “universe of discourse” to another.
31[Wittgenstein, 1974], p. 17.
generality. The main conclusion that can be drawn from 5.501 is that essential generality is fully captured by form-series generality only.

The extensional determination of the range of values of a variable does not make sense as such (except maybe in the case of a finite extension, i.e., in the case of an enumeration) because the reference to an extension, without any rule to range and survey it, does not deduce any properly logical specification at all. A variable sign lacks any mode of signification and remains logically undetermined if the task of determining the identity and number of the objects which satisfy some propositional function is delegated to the world instead of residing in the symbol itself. Of course, if one says “There is a pen somewhere on this table,” some generality is actually expressed in a perfectly meaningful way. However, such a kind of generality has nothing to do with logical generality—the kind of generality that is expected and required in the case of logical truths. This diagnosis, which goes beyond Wittgenstein’s hostility toward set theory, explains the stress put by Wittgenstein on induction throughout his later reflection on mathematics. Inductive generality, drawing on form-series generality, became the paradigmatic figure of logically controlled, non enumerative generality at the time Philosophical Grammar was written: There are, on principle, as many kinds of generality as there are discursive “systems,” but this does not detract from the fact that the iterability of an operation constitutes the core of mathematical generality. This is what can already be found in the Tractatus. As underlined at 5.501, the variety of notational systems to express generality remains an important feature of language, which no essential form of generality should completely obliterate; However, there does exist an essential form of generality, namely form-series generality.

So the conclusion is that the “essential” general validity of logical propositions has to have to do with some series of forms. It remains to be seen which.

3 Essential general validity

Let us consider \( p \supset q \supset q \). There is no question that this proposition is both generally valid (owing to some sort of schematical generality) and necessarily true as well. It remains true however ‘\( p \)’ and ‘\( q \)’ are replaced with other propositions,

32[Wittgenstein, 1961], 23.10.14, p. 17: “If the completely generalized proposition is not completely dematerialized, then a proposition does not get dematerialized at all through generalization, as I used to think.”

33[Marion, 1998], p. 98 sq.

34[Wittgenstein, 1978], pp. 458-459: “How a proposition is verified is what it says. Compare generality in arithmetic with the generality of non-arithmetical propositions. It is differently verified and so is of a different kind.”

35[Wittgenstein, 1978], p. 457: “What is general is the repetition of an operation.”
and it remains true in any possible situation whatsoever as well. The real question is how are we to represent these two features, and understand their combination? 6.1231-6.1232 clearly mentions both generality and essential validity (as opposed to “accidental” validity or truth “as a result of a fortunate accident”). Now, one thing is clear: The essentiality that the Tractatus ascribes to the general validity of logic comes not only from an essential (i.e., non-accidental) validity, but also from an essential generality. Essential general validity, indeed, involves some generality, yet it follows from the above that a mere generalized (i.e., universally quantified) proposition cannot have the essentiality required by logical truth: Generalization cannot mean but a general fact. So the non-accidentality of the general validity of logic also requires the non-accidentality of the general validity of logic.

How, then, to combine essential generality and essential validity so as to get the essential general validity that 6.1232 assigns to logical propositions?

The question is legitimate, because generality and validity point to different directions. Indeed, validity has to do with truth-functionality, and 5.521 insists precisely on the separation to be made between generality and truth-functionality. This separation is what the operator N is meant to bring out.

Let us consider this latter point. Fogelin has deemed the operator N to be inadequate to indicate the respective scopes of variables and, as a consequence, to capture quantifier alternation (i.e., to express mixed quantified formulae such as \((x)(\exists y)fxy\) or \((\exists x)(y)fxy\)). This assessment sparked a very substantial literature, devoted to evaluating the possibility of recovering first-order logic from the operator N. Along with the issue of the operator N’s expressive sufficiency, the question has also been raised as to whether the Tractatus is committed (by 6.113) to a decision procedure for checking that a given proposition is a logical truth. Although those questions are interesting by themselves, the introduction of N does not purport to replace the syntax of quantification, let alone to improve on it. This is most definitely borne out by 4.0411. The whole rationale of the operator N is to symbolize verifunctionality in the most general way and to distinguish as clearly as possible what 5.521 says that Frege and Russell failed to clearly distinguish, namely generality (i.e., the introduction of a class of propositional values) and verifunctionality (i.e., a certain combination of the corresponding truth values). The logistic functional notation precisely conflates both components: A propositional function is supposed to simultaneously select a domain of arguments, assign a value to each of them, and turn out the logical product (or the logical sum) of all values. In contrast, the notation ‘\(N(\bar{\xi})\)’ introduced at

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36[Fogelin, 1976], particularly pp. 78-79.
38On this question specifically, see [McGray, 2006], pp. 161-168.
39This point is well brought out in [Sackur, 2005] (pp. 129-137). See also [McGinn, 2006], pp.
5.501-5.502 is geared to represent, in the most general and perspicuous way, the combination of two distinct factors represented as distinct: the determination of a variable, which consists in the selection of a system of propositions, as represented by $\xi$; and all the possibilities of truth-functional calculus, as represented by $N$. The separation of both factors is the condition of their combination, as opposed to their confusion.

Form-series generality and truth-functional validity thus refer to independent propositional structures. Essential general validity, however, corresponds precisely to the case where they become indistinguishable. How to understand this?

Indeed, if one stresses the sole generality of a tautology such as $p \rightarrow q \rightarrow q$, one is led to describe it in terms of quantificational generality (understood substitutionally or otherwise), in the sense of its being made fully explicit as $(\phi) \cdot (\psi) : \phi \rightarrow \psi : \rightarrow \psi$. However, this is clearly incompatible with what Wittgenstein has in mind:

[…] Wittgenstein believes that we must be careful to distinguish the general proposition of logic from generalizations of material propositions. On his view, construing $(p)(p \lor -p)$ as a substantive general truth about logical objects, obscures this distinction.

On the other hand, stressing the sole truth-functional validity of a tautology amounts to conceiving the latter as a truth that remains true in every possible world, and this is equally misleading, because it clearly presupposes (and only rephrases in a figurative way) what is to be analyzed. A logical proposition is a proposition whose truth can be established on the basis of its symbol alone. This has nothing to do with evaluating it in various possible worlds so as to bring out some kind of invariance. By way of analogy, logical truth is to truth across all possible worlds what “direct reference” is to “obstinately rigid designation,” to use David Kaplan’s terminology. An obstinately rigid designator is a term which designates the same individual object in every possible world (whether that object exists in that world or not). Kaplan takes a directly referential expression to be rigid in the deeper sense that its referent, once determined, is taken to be fixed for all possible circumstances because it is the propositional component itself:

231 and 235-236.

[McGinn, 2006], p. 60. See also p. 248: “A particular instance of a proposition of this form [namely, $(p \lor -p)$] is not a logical truth in virtue of being a substitution instance of a general logical law, $(p)(p \lor -p)$, but simply in virtue of the way it is constructed, that is, simply in virtue of its having the form $(p \lor -p)$.” Ricketts ([Ricketts, 2013], p. 127) also insists on the distinction to be made between a relation between entities, on the one hand, and the construction of a logical structure such as a material conditional, on the other. Consequently, logical generality cannot consist in the substitutability of entities, but only in the generality of a construction, i.e., in the iterative generativity of a truth-operation.

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For me, the intuitive idea is not that of an expression which turns out to designate the same object in all possible circumstances, but an expression whose semantical rules provide directly that the referent in all possible circumstances is fixed to be the actual referent.\footnote{[Kaplan, 1989], p. 493.}

Kaplan explains the rigidity of proper names as a consequence of their being directly referential:

If the individual is loaded into the proposition (to serve as the propositional component) before the proposition begins its round-the-worlds journey, it is hardly surprising that the proposition manages to find that same individual at all of its stops, even those in which the individual had no prior, native presence. The proposition conducted no search for a native who meets propositional specifications; it simply ‘discovered’ what it had carried in.\footnote{[Kaplan, 1989], p. 569.}

In Kaplan’s view, direct reference is the deep semantic structure explaining (obstinately) rigid designation. In the same way, tautologyhood represents in the \textit{Tractatus} the deep symbolic structure that explains the invariance in truth value of a logical proposition.\footnote{This point dovetails with Diamond’s remarks quoted in footnote 10. As Sanford Shieh summarizes: The reason why a tautology is true in a special way “is not because it describes a special type of invariably obtaining situations. [. . . ] it is the nature of linguistic representation, rather than features of the world or of all possible worlds, that makes tautologies true.” ([Shieh, 2011], p. 3)} Just as it is guaranteed in advance of evaluation that a directly referential term has one and the same referent in all possible worlds, analogously it is guaranteed in advance that a logical proposition remains true across all possible worlds. Logical propositions pertain to everything that is essential to a proposition’s being true or false,\footnote{As Wittgenstein puts it in his \textit{Notebooks}: “The logic of the world is prior to all truth and falsehood.” ([Wittgenstein, 1961], 18.10.14, p. 14)} so the notion of possible world (defined as a certain assignment of truth values) is derivative of that of logical truth—which is why one cannot resort to the former to explain the latter.

Thus, although generality and truth-functional validity correspond to independent structures, the special validity peculiar to logical propositions requires more than their external combination, as in the case of regular general propositions: It requires their coincidence. So there remains only one option: The series of forms underpinning the form-series generality of a given logical truth has to be identical with the series of forms underpinning truth-functionality in general. Let us try to expand upon this idea.

At 6, Wittgenstein identifies the general form of a proposition with the general form of a truth function, as given by the series of forms \([\bar{p}, \xi, N(\xi)]\). As
emphasized by 6.001, this general form means that any proposition can be exactly identified with the series of forms (starting from certain elementary propositions) that leads to it. In the same way, a tautology is nothing else but the drawing of the “truth-diagram” showing that no combination of truth values can lead to the pole written F, as shown at 6.1203 in the case of \( \sim(p, \sim p) \). The general validity of a tautology follows from its method of constructing only. In the case of \( p \cdot p \supset q \cdot q \cdot q \cdot p \) and \( q \) stand for any propositions: They act as mere indices of truth values. However, this holds exclusively in the context of the tautologyhood of the whole proposition. The general validity of the tautology certainly cannot be rendered by \( (p) \cdot (q) \cdot p \cdot p \supset q \cdot q \cdot q \), because the generalization over \( p \) and \( q \) makes sense only after the tautologyhood of the proposition has been granted, precisely. As soon as the tautologyhood of the proposition is established, however, it becomes obvious that \( p \) and \( q \) can be replaced by whatever propositions one may wish. The general validity of a tautology is thus internal to the propositional symbol that one wishes to generalize over. That is the reason why it eludes any generalization (in the form of a universal quantification), as claimed by 6.1231. In other words, the truth-diagram of \( p \cdot p \supset q \cdot q \cdot q \cdot p \) is integral to the full symbol of that tautology, and its method of constructing shows, as an essential feature of it, how ‘\( p \)’ and ‘\( q \)’ are thereby neutralized. This is the “zero-method” mentioned by 6.121. Hence, the general validity of a tautology is essential both because its validity is internal to its symbol and because its generality is internal to its validity (being neither its ground nor its external outcome).

To sum up, in the symbol-directed approach to both logical generality and logical necessity that is pursued by the *Tractatus*, the generality as well as truth-functional invariance of a logical proposition are built into its symbol. As a result, it is meaningless to express the generality through some universal quantification or some substitution rule. The universal quantification or the substitution rule can be but derivative and inaccurate ways of speaking. For the same reason, it is meaningless to describe the proposition as remaining true however the truth-value assignment is changed, as if the proposition were attached to a particular original assignment to begin with.

The series of forms \([\tilde{p}, \tilde{\xi}, N(\tilde{\xi})]\) is not exhausted by its role of presenting the general form of a proposition as a truth function. Landini objects to Anscombe’s identification of the general form of a truth function, \([\tilde{p}, \tilde{\xi}, N(\tilde{\xi})]\), with the general term of a consecutive series of truth functions, \([\tilde{p}, N^n(\tilde{p}), N^{n+1}(\tilde{p})]\)\footnote{[Potter, 2009], pp. 160-164.}.

Wittgenstein’s assertion that all truth-functions can be reached by “successive applications” of the N-operator does not require Anscombe’s identification of his notion of “the general form of a truth-function”\footnote{[Anscombe, 1959], p. 132.}.
with the notion of the *general term* of a *consecutive* series of truth-functions. When Wittgenstein said that every truth-function is the result of successive applications of the N-operator, he may have simply meant that the N-operator is expressively adequate.\(^{47}\)

Accordingly, the general term corresponding to the general form of a proposition is not meant to represent all propositions as belonging to a single ordered series (with *all* elementary propositions as its basis) but rather, more flexibly, any particular truth function (with only the relevant subset of all elementary propositions as its basis: the set of those elementary propositions that occur in its constructional history).\(^{48}\) This fits Wittgenstein’s use of the bar (at 5.501) as standing for the determination of a certain selection: a certain selection of the bases in the case of \(\bar{p}\), or a certain selection of propositional values in the case of \(\bar{\xi}\).\(^{49}\)

This flexible adjustment of the notation \([\bar{p}, \bar{\xi}, N(\bar{\xi})]\), depending each time on the particular truth-functional setting at stake, allows one to understand how the general form of a truth function can coincide with the series of forms ruling the generality of a particular logical proposition. Indeed, given a logical proposition \(\Theta(\bar{p})\) (for instance, \(\bar{p} = \{P, Q\}\) in the case of \(\Theta = P \supset P \supset Q\)) one can express both its essential generality and its essential validity by writing:

\[
\Theta([\bar{p}, \bar{\xi}, N(\bar{\xi})]) = [\Theta(\bar{p}), \Theta(\bar{\xi}), \Theta(N(\bar{\xi}))].
\]

The left-hand side of this identity stresses the truth-functional validity of \(\Theta\), whereas its right-hand side stresses the form-series generality of \(\Theta\). Indeed, the expression on the left provides the truth-functional calculation that takes place when elementary propositions selected within the basis \(\bar{p}\) of \(\Theta\) are replaced with their respective negations, in other words when the assignment of truth values to elementary propositions of \(\Theta\) is changed, and generally when the truth value of \(\Theta\) is considered for all possible truth-value assignments. As to the expression on the right, it implements the general form of a proposition within \(\Theta\), which corresponds to the fact that the basis of \(\Theta\) can be replaced by any appropriate selection of propositions: all occurrences of ‘\(P\)’ replaced by any proposition \(\phi\) and similarly for all the other members of \(\bar{p}\). This is actually what the end of 6.124 conveys: “If we know the logical syntax of any sign-language, then we have already been given

\(^{47}\)[Landini, 2007], p. 143. See also [Ricketts, 2013], p. 140: “[…] the general form of sentences has an intrinsically schematic character. […] The general sentence-form is rather a scheme for the construction of any sentence: it is the most general form for the construction of truth-functions of elementary sentences.”

\(^{48}\)In the series \([\bar{p}, \bar{\xi}, N(\bar{\xi})]\), contrary to what McGinn claims ([McGinn, 2006], p. 234), \(\bar{p}\) does not have to be the totality of all elementary propositions.

\(^{49}\)This point is recalled by [Landini, 2007] (p. 142).
all the propositions of logic,\textsuperscript{50} which means conversely that any proposition of logic already carries the whole logical syntax of all propositions.

Of course, both sides of the identity above correspond to the same thing, namely the general validity that 6.1232 seeks to bring out. The series of forms that gives the general form of a truth function and corresponds to the truth-functional invariance peculiar to tautologies is at the same time the series of forms that grounds the essential generality peculiar to logical propositions. This coincidence of the two series of forms is finally what explains why the essential generality and the essential validity of the propositions of logic merge into a single essential general validity.

It is now possible to get back to the starting point of sections 5.501 and 5.521, which convey a criticism of both Frege and Russell. Two distinct kinds of generality run in parallel in Frege’s \textit{Begriffsschrift}: quantificational generality, as expressed by means of individual variables, and schematic generality, as expressed by means of schematic letters for judgeable contents (any particular judgeable content being substitutable for a schematic letter). Both kinds, however, are combined, as illustrated for instance by the proof of judgment (93), where both substitutions $(a \mapsto f, f(\Gamma) \mapsto \Gamma(y))$ are carried out.\textsuperscript{51} Wittgenstein implicitly objects to this ambiguity. There are two symmetric ways to clear it up: either by considering schematic letters as implicit variables (this is the Russellian way, with the delicate distinction between \textit{entity-variation} and \textit{meaning-variation})\textsuperscript{52} or by tracing propositional variables back to schematic form-series generality: This is the Tractarian way.

The second criticism that can be found at 5.521 is about Russell. Wittgenstein’s target is not only the quantificational symbolism of \textit{Principia Mathematica} but also, maybe primarily, the substitutional theories that Russell developed between 1905 and 1908 and whose main published expression is “On ‘Insolubilia’ and Their Solution by Symbolic Logic.” Russell’s framework is based on substitution, i.e., on the replacement of some constituent of a given proposition by another entity. For a proposition $p$ and a constituent $a$ of $p$, the expression ‘$p/a; b \models q$’ is taken as primitive, and means that $q$ results from replacing $a$ by $b$ in $p$. If $p$ is a proposition, ‘$(x): p/a; x \vdash q, &. q$’ then means that a replacement of $a$ by $x$ in $p$ results in a true proposition, for any $x$ whatsoever.\textsuperscript{53} The main point of the Tractarian criticism here is that the formal substitutability of $b$ for $a$ in $p$ indicates the substitutability of any entity for $a$, and thus a variable argument: Generality is already present. In Russell’s substitutional logic, the substitution of the entity variable $x$ for $a$ has exactly the same status as the substitution of $b$ for $a$. However,

\textsuperscript{50}[Wittgenstein, 1974], p. 63.
\textsuperscript{51}[Frege, 1972], p. 183.
\textsuperscript{52}See [Russell, 1992b], pp. 360 ff.
\textsuperscript{53}See [Russell, 1973], pp. 200-201.
in Wittgenstein’s view, the generality of $x$ substituted for $a$ already lies in the very substitutability of $x$ for $a$. Russell makes it appear as if the generality carried by $x$ were magically attached to $x$ independently of this substitutional context and could simply be added to the operational apparatus of substitutions. Against this presentation, Wittgenstein claims that the substitutability of expressions (propositions) to others already appeals to some form of generality: ultimately, to form-series generality.

In conclusion, the essentiality of the general validity that the *Tractatus* attributes to logic is finally explained. This validity is described by the *Tractatus* as “essential” because logical generality and logical validity are both thought of as being essential, and as being essential to each other. Essential generality is the generality expressed by essential features of a propositional symbol (3.34), and corresponds to what has been called form-series generality (5.501). Essential validity is the validity of a proposition whose truth can be determined by inspection of its symbol alone (6.113), and corresponds to tautologyhood (5.525). Essential generality and essential validity go together because they rely on the same series of forms, the one that presents the general form of a truth function. Owing to that form, which is a variable whose values are all the propositions, the elementary propositions involved in the construction of the truth-diagram of a tautology acquire by the same token a schematic generality that makes any proposition substitutable for each of them, and makes it possible in return to recognize a logical form as such.

**References**


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