Logic and Ontology

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Ontology = theory of everything that is (and of the different kinds of beings which are part of everything that is).

Logic = theory of what makes it possible to speak of everything.

This is already the case in Kant's *Critique of pure reason*. About Transcendantal Analytic as pertaining to the form of a possible experience, Kant writes:

> *Its principles are merely rules for the exposition of appearances; and the proud name of an Ontology that presumptuously claims to supply, in systematic doctrinal form, synthetic a priori knowledge of things in general (for instance, the principle of causality) must, therefore, give place to the modest title of a mere Analytic of pure understanding. (A247)*

Kant remains an important reference to analytical philosophy, but the way the universe of things in general is conceived of has completely changed since Kant.

Jean-Michel: Tarski's semantics has turned the universe of set theory into a substitute for the world.
Every possible configuration of the world can be represented as a set-theoretic model.

That's where mathematics properly speaking enters the scene of analytical philosophy of language.

In Kant, the problem is to not include too much, so as to delineate the proper domain where objective knowledge is possible. The objective generality of transcendantal logic should not be confused with the abstract generality of formal logic.

In a reversal from Kant, the issue at stake in analytic philosophy is how to include as much as possible, and not to leave anything aside. The target is **absolute generality**.

Absolute generality is the feature of any discourse whose scope purports to embrace absolutely everything. It runs across, or is supposed to run across logical paradoxes, the first of which being Russell's paradox. Here is how the predicament is set up:

- Logic has become first-order logic
- The interpretation of first-order logic, carried out by Tarski's semantics, relies on the set-theoretic universe.
- Set theory becomes inconsistent if one supposes that there is a universal *set*.
- Hence, there is no way to genuinely refer to absolutely everything.

That is why absolute generality has become, in analytical philosophy, a central issue.
For many years since Russell's paradox, absolute generality has been considered as doomed to inconsistency.

The whole matter has been gradually reconsidered since the 1990's.
Russell's paradox has been proven not to make universality absolutely unpracticable. (Set theory has to be changed, instead of absolute generality being renounced.)
On the other hand, generality relativism (the philosophical stance according to which generality is always restricted, in a context-relative way) is not satisfying.

General remarks:

- SPEAKING OF EVERYTHING DOES NOT GO WITHOUT SAYING. The possibility to speak consistently of everything has to be accounted for. The stress put on that preliminary question singles out analytical philosophy.
- The issue of absolute generality lies at the crossroad of set theory and ontology. This is typical of modern logic. Absolute generality is a central example of the way in which the mathematical resources of logic help to assess the constraints set upon absolute generality and to conciliate them as much as possible with the demand, coming from philosophy, that absolute generality should be available.
- A secondary topic brought up by the issue of absolute generality bears on the ontological status of modal entities: are possible entities part of everything that is? (For lack of time, I will not broach that question.)

Summary

1. **The logical universalism in question**
2. **Russell's paradox and set theory**
3. **The issue of absolute generality**

The logical universalism in question

Logical universalism = family of philosophical stances expressed by Frege, Russell, Wittgenstein and, to some extent, Carnap or even the young Tarski, and which have in common to uphold the strict universality of logic.

Logic is traditionally conceived of as a theory that precedes any other theory. Such a precedence consists in the unconditional bearing of logic on thought in general.
This characterization of logic has been cashed out in terms of two distinct features:

- the **universality** of logic
- the **radicality** of logic

The universality of logic consists in its being about absolutely everything : only one type of entity variables ; a single unrestricted range of values for entity variables. **Nothing can escape logic.**

The radicality of logic corresponds to there being the one and the same logic that any reasoning must comply with. The principles of logic are laws that one cannot but presuppose them, even to challenge them. **Nobody can escape logic.**

The universality and the radicality of logic have been associated since at least Kant's *Logic*:

> *If [...] we put aside all cognition that we have to borrow from objects and merely reflect on the use just of the understanding, we discover those of its rules whih are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. (Jäsche logic, Ak.* IX*, 12)*

It is, as well, "a science a priori of the necessary laws of thought, not in regard to particular objects, however, but to all objects in general" (Ak. IX, 16). (Universality)
Logic is the "science of necessary laws of thought, without which no use of the understanding or of reason takes place at all" (Ak. IX, 13). (Radicality)

Frege himself gives in sometimes to a kind of confusion:

> *Here, we have only to try denying any one of them [one of the fundamental truths of arithmetic], and complete confusion ensues. Even to think at all seems no longer possible. The basis of arithmetic lies deeper, it seems than that of any of the empirical sciences, and even than that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought? (Foundations of Arithmetic, § 14)*

Asymmetry of Kant's and Frege's respective agendas.

To Kant, radicality is a way of accounting for formal universality from within a nonformal conception of logic.
The domain ruled by formal logic can absolutely not be seen as more inclusive than the domain ruled by trancendental logic.
The generality of formal logic has to be reckoned through something else than its universality, namely: it states conditions of possibility of thought as such.

Frege's move goes the other way around: the universality of logic allows Frege to express the radicality of logic from within an anti-psychologistic conception of logic, without committing himself to some slippery notion of necessity.

To sum up: Both Kant and Frege bestow on logic two distinctive features: universality and radicality.

And both agree to consider those features as inseparable.

But Kant puts the stress on radicality, whereas **in Frege logical universality prevails over logical radicality**.
The same holds about Russell, who considered modal notions as meaningless, and defined logic by its universality.

According to Frege and Russell, logic is both *the* absolutely general theory, and the theory *of* absolute generality.

"Logical absolutism" = conflation of logical universality and logical radicality, in addition to the further claim that "one cannot consider one's language from the outside."

Although a label that has been pinned on to Frege or Russell, logical absolutism is *not* epitomized by Frege and Russell themselves.
Logical absolutism is more of an invention by its enemies, i.e. by those (the late Tarski, Heijenoort, Hintikka, . . .), who fought the quest for universality.

A tradition initiated by Jean van Heijenoort put in the same "absolutist" bag four independent claims:

- logic resorts to absolutely unrestricted (unspecified) variables
- logic refers to one single universe of discourse
- the language of logic is unescapable
- meta-theoretical studies, and semantics to begin with, are impracticable and, for that reason, banned.

Hintikka about the "ineffability of semantics:"

> *One of the main consequences of the universality of language (universality of the language) is that I cannot in my language speak of how its semantical relations to the world could be changed, at least not in a large scale. But such a systematic variation of the interpretation of a language is what the model theory for this language is all about. To speak of different models of a theory or a language in a logician's sense is ipso facto to speak of different systems of referential relations (interpretations) connecting language (or a part thereof) with the world. Hence all model theory is impossible according to the strict constructionist version of the universalist assumption.*
> *(Lingua Universalis vs. Calculus Ratiocinator: An Ultimate Presupposition of Twentieth-Century Philosophy, 1997, p. 216)*

Hintikka again:

> *A believer in the free reinterpretability of our language would answer a question concerning the range of an existential quantifier by saying, ‘Whatever we have included in the relevant universe of discourse.’ For a universalist, there is only one range for one’s (first-order) quantifiers, viz., all the individual objects in the world.*
> *(Lingua Universalis vs. Calculus Ratiocinator, 1997, p. 218-219)*

Hintikka assimilates the quest for universality (universal scope of logical variables), and the quest for a “universal medium” in which one can say everything and in which one must say anything.

Several scholars (Jamie Tappenden, Richard Heck, Gregory Landini) showed that there was no objection of principle, neither in Frege nor in Russell, against "meta-theoretic" considerations. The universalist does not even balk at considering local universes of discourse.

The confusion of the universality of logic and of the "unescapability" of the language of logic does not come from the authors who are supposed to epitomize logical universalism.

Origin of the confusion: Tarski's 1935 essay on "The concept of truth in formalized languages."

Tarski: An adequate definition of the truth predicate of some language can only be given in a metalanguage whose order is strictly greater than the order of the language under consideration.
This excludes the colloquium language, because of its universality$_T$ (universality in Tarski's sense: whatever can be expressed in whatever language, can be expressed in the colloquium language).
The order of a language is defined as the maximal order of all its semantical categories, and the colloquium language, being "universal," is of infinite order.

However, as Rouilhan has pointed out, Tarski replaced, in the post-scriptum, the original notion of order with a new one, drawn from the cumulative hierarchy of ZF set theory. The order of a variable (and, consequently, the order of a language) is redefined by the set-theoretical rank of its possible values.

Thus, the order of a language becomes the least upper bound of the ranks of all sets whose existence follows from the axioms. This change precipitated a set-theoretical conception of the hierarchy of languages: the classes to which refer the expressions of the object-language must all belong to classes to which only expressions of the metalanguage refer.

So the domain of the metalanguage must be strictly wider than the domain of the object-language.
As a consequence, a universal domain cannot lend itself to any metalinguistic truth theory.
Hence the confusion of the universality of the variables of a language with the universality$_T$ of this language (in Tarski's sense), and thus with its "unescapability."

But the confusion only follows from an artefact of Tarski's analysis.

Absolute generality = intended unrestricted scope of the variables used by the language of logic.

Absolute generality as such has to do neither with the universality$_T$ of ordinary language, nor with the "unescapability" of the language of logic, nor again with the radicality of logic.

The issue of absolute generality is mainly:

logical universalism *vs* logical particularism.

Logical particularism = there are but particular, limited "universes of discourse" (*cf.* Tarski's semantics and model theory).

To sum up:

- The logical universalist claims logic to be completely universal.
- The logical particularist: “No, logical universality is an illusion, there are but particular universes of discourse. Logical universalism is wrong, because it is absolutism: it turns language into a prison.”
- In fact, the intended universality of logic (= absolute generality) is NOT the universality$_T$ of ordinary language, and logical universalism is NOT logical absolutism.
- The logical particularist: “Still, logical universalism is wrong, because logical universality is wrong, because Russell’s paradox proved it to be inconsistent.

Let’s see.

Russell’s paradox and set theory

Russell’s paradox by Russell. *Principles of mathematics* (1903), SS103-104 & App. B.

§101: Russell’s paradox is set out in terms of predicates, in terms of class concepts and in terms of classes.

- The predicate “not-predicable of oneself” is predicable of itself iff it is not.
- The class-concept “Class-concept which is not a term of its own extension” is a term of its own extension iff it is not.
- *A class as one may be a term of itself as many. Thus the class of all classes is a class; the class of all the terms that are not men is not a man, and so on. Do all the classes that have this property form a class? If so is it as one a member of itself as many or not? If it is, then it is one of the classes which, as ones, are not members of themselves as many, and vice versâ.*

Russell's paradox in modern notation

Let $w = \{x : x \notin x\}$.
By definition of w, one has, for any thing x :

$$x \in w \text{ iff } x \notin x.$$

Plugging w into x, one gets:

$$w \in w \text{ iff } w \notin w. \text{ Contradiction.}$$

Conclusion drawn by Russell: the classes which as ones are not members of themselves as many, do not form a class as one.

Hence the general question:

> *Which propositional functions define classes which are single terms as well as many, and which do not? And with this question our real difficulties begin. (Principles, §102)*

A way out? The distinction of logical types.

> *A class as one, we shall say, is an object of the same type as its terms; i.e. any propositional function* $\phi(x)$ *which is significant when one of the terms is substituted for* x *is also significant when the class as one is substituted. But the class as one does not always exist, and the class as many is of a different type from the terms of the class, even when the class has only one term [. . .]. And so "x is one among x's" is not a proposition at all if the relation involved is that of a term to its class as many [. . .]. It is the distinction of logical types that is the key to the whole mystery. (§104)*

In other words, $x \in x$ amounts to:

> x-as-one is one among all the members of x-as-many.

But x-as-one and x-as-many are of different types, and thus cannot be identified.

The Appendix B formalizes types in a starker way.

A *type* is the “range of significance” of some propositional function.
Types make up a *simple hierarchy*:

- type of the individuals
- type of the classes of individuals
- type of the classes of classes of individuals
- ...
- type of the relations
- type of ranges of relations
- type of relations of relations
- ...
- ...

Simple Type Theory: Only

$$x^{(n)} \in y^{(n+1)}$$

is permissible.

So "$x \in x$**" is meaningless**, because x cannot have two different types. Accordingly, $x \notin x$ is meaningless as well.

Appendix B (§§497-498):

- All ranges of significance form a range of significance, "consequently $x \in x$ is sometimes significant."
- All objects do form a type, since every object is identical with itself. So THERE IS STILL A UNIVERSAL CLASS.

Despite appearances, Russell's type theory is **NOT** a way of reneging on the universality of logic.

Naive Set theory (Cantor)

Naive conception of sets : A set is the extension of some property.
Naive comprehension principle: for any property $\mathcal{P}$, there exists a corresponding set $E_{\mathcal{P}} = \{x : \mathcal{P}(x)\}$.

Consider the property $\mathcal{R}$ of "not belonging to oneself" ($\mathcal{R}(x)$ is "$x \notin x$").
The comprehension principle gives then the set $E_{\mathcal{R}}$, which is nothing but the contradictory class w of Russell's paradox.

So Naive Set Theory is inconsistent.

Zermelo-Fraenkel Set Theory (ZF)

It is a formal (first-order) theory. Its axioms are:

- Extensionality axiom: $\forall z(z \in x \leftrightarrow z \in y) \rightarrow x = y$.
  (If two sets x and y have the same elements, then they are identical.)
- Pairing axiom : $\forall a\, \forall b\, \exists c\, \forall x\, (x \in c \leftrightarrow (x = a \vee x = b))$.
  (Given any two sets a and b, there is a set c which contains exactly a and b)
- Separation scheme: For any formula $\varphi(x)$ and any set x, there exists a set y such that $\forall z\, (z \in y \leftrightarrow (z \in x \wedge \varphi(z)))$.
  (The set y contains exactly all the members of x that have the property expressed by φ.)
- ...

Reductio ad absurdum. Suppose that there is a universal set V. Then the separation scheme guarantees that there exists $V' = \{x \in V : x \notin x\}$. But V' is nothing but the contradictory class w of Russell's paradox. So there is no universal set.
ZF turns Russell's paradox into the theorem THAT THERE IS NO UNIVERSAL SET.

Tarski's semantics (and logical particularism) starts from there. ABSOLUTE GENERALITY IS MATHEMATICALLY FLAWED.

Should we surrender?

"On Some Difficulties in the Theory of Transfinite Numbers and Order Types" (1905): **The separation axiom has to be sacrificed, not the existence of a universal class !**

Cantor's Theorem:

$$\forall u \, \forall y \, (y \subseteq u \to y < \wp(u))$$

This theorem is also supposed to prove the impossibility of a universal set V. Indeed, for $u = V$ and $y = \wp(V)$, one has $\wp(V) \subseteq V$, and as a result $\wp(V) < \wp(V)$. Contradiction.

BUT, as Russell points out, Cantor Theorem precisely does not apply in the case where u is taken to be V.

Indeed, Cantor's proof draws a contradiction from the assumption that there exists a map $g : u \to \wp(u)$ such that $g(u) = \wp(u)$.

If such a map were to exist, Cantor argues, then the set $C = \{x \in u : x \notin g(x)\}$ could not be in $g(u)$, contradicting the assumption about g.

BUT, for $u = V$, C is nothing but the contradictory class $\{x : x \notin x\}$ of Russell's paradox. So C cannot be invoked.

Russell's conclusion: Cantor's proof relies on a wrong application and does not work for $u = V$.

Otherwise put, set theory begs the question and postulates that no universal set can be introduced.

In Russell's mind, on the contrary, the existence of the universal class $V = \{x : x = x\}$ is self-evident.

So one reaches stale mate.

History of logic and mathematics has sided with set theory against Russell, but this is no philosophical justification.

Admittedly, Cantor's naive abstraction principle leads to a contradiction.
But that does not confute as such the Russellian conception of sets as extensions (of propositional functions): Some extensions cannot be sets, but that does not detract from Russell's contention that all sets are extensions.

Conclusion: Abandoning Russell's way is not forced upon us by Russell's paradox.
Shifting to ZF set theory (and Tarski's semantics) is not mandatory.
There is more leeway that one usually makes out to be.

This is exactly what is proved by NF set theory.
("NF" stands for 1937 Quine's paper, "New Foundations for Mathematical Logic").

The main idea is to maintain the naive comprehension principle, but to balance it out by building a simple type theory into the syntax of the language.

Variables are typified, and a stratified formula is a one that abides by the rules of simple type theory for its variables. For instance, $x^n \in y^{n+1}$ is stratified, but $x^n \in x^n$ is not.

The axioms of NF are:

- Extensionality:
  $\forall x^{n+1} \forall y^{n+1} \, [x^{n+1} = y^{n+1} \leftrightarrow \forall z^n (z^n \in x^{n+1} \leftrightarrow z^n \in y^{n+1})]$
- Comprehension for all stratified formulae:
  $\forall x \, \exists y \, \forall z \, (z \in y \leftrightarrow \phi(x, z))$
  where ϕ is stratified (and y is not free in ϕ).

Since $x = x$ is stratified, there exists a universal set V (by comprehension). Thus $V \in V$, but no paradox ensues, since V is not a variable.

As Quine says about NF:

> *The variables are to be regarded as taking as values any objects whatever; and among these objects we are to reckon classes of any objects, hence also classes of any classes.*

The issue of absolute generality

Richard Cartwright ("Speaking of Everything", *Noûs*, 1994): Cartwright assumes as a matter of fact the universality of ordinary quantification ("Everything is mortal if human") — and thus the availability of a first-order language the variables of which range over everything there is.

> *[. . .] we are apparently to understand that the set-theoretic paradoxes somehow show that the variables of a first-order language cannot range over all objects whatever.*
> *The disputed proposition, the one I propose to defend, is this: any objects there are can simultaneously be the values of the variables of a first-order language. (Cartwright, p. 2)*

Cartwright, cont'd:

> *There is no set that has as members all and only those things that are not members of themselves. But the things that are not members of themselves can simultaneously be the values of the variables of a first-order language; so at any rate I claim. [...]* **for in order that certain objects be the values of the variables of some first-order language, it is on my view not necessary that there be some one object of which they are the members***. Anyhow, the assumption could not be invoked across the board: there is nothing the members of which are all and only those things that are not members of themselves. (Cartwright, p. 3)*

Cartwright, cont'd:

> *It is consistent with the view I want to defend to speak of the universe of discourse of a language. [...] But it does involve a certain risk, the risk of being understood to imply that the universe of discourse is an object — a set, or class, or collection — of which the values of the variables of the language are the members. The implication must simply be disavowed: to say that the universe of discourse of a language comprises the ordinal numbers is to say no more than that the ordinal numbers are the values of the variables of the language. [There is no further set of those values.]*
> *(Cartwright, p. 3)*

If we quantify over classes, this DOES NOT IMPLY that we have the collection of all classes to talk about.

The general idea that comes in for Cartwright's criticism is what Cartwright calls "the All-in-One Principle:"

> *The general principle appears to be that to quantify over certain objects is to presuppose that those objects constitute a "collection," or a "completed collection" — some one thing of which those objects are the members [in other words, the set domain of a universe of discourse].*

Conclusion: The unviability of absolute generality comes only from an artefact of Tarski's semantics, namely: the values of any variable have to make up a set, that in some way precedes those values.

Timothy Williamson, “Everything” (*Philosophical Perspectives*, 2003).

Tarskian semantics interprets any universal quantification as restricted to the domain of a given set-theoretic structure (a.k.a. “universe of discourse,” or “interpretation of the language”). “Any x” is each time interpreted as “any member of the domain D,” for some domain D.

Each such domain is a certain model of *everything*, but there is no way of having *really* everything. (The set-theoretic universe, according to ZF, is not a set. It can be a domain only for the meta-language.)

Such is the unsettling situation that Williamson is unsatisfied with.

Williamson first sets out an argument, a kind of generalization of Russell's paradox, to the effect that all the interpretations of a formal language L cannot be members of a single domain.

Semantical reflection about a language L necessarily gives rise to an extension of the intended interpretation of L.

This is true about any language, the metalanguage ML included, so that "generality-absolutism" (as Williamson calls it) is not consistent.

Williamson's paper is primarily meant to show that generality-relativism does not fare any better.

Tarskian semantics:

$\forall x\, Fx$ ["Everything *F*s"] is true in an interpretation *M* with domain *D*
iff
any $d \in D$ satisfies *F* in *M*, i.e., iff **everything** satisfies *F* as soon as it belongs to *D*.

The latter clause has to be read unrestrictedly.

The domain *D* of the interpretation acts as a contextual restriction, but eventually drives back to absolute generality.

Hence the following predicament:
When I refer, in the meta-language ML, to "any $d \in D$"

- either I do not refer to absolutely everything, but then the semantics of L is not the intended semantics;
- or I do in fact refer to absolutely everything, but then absolute generality is available to ML.

Williamson's point (p. 427) is about the following claim made by the generality-relativist:

(5) It is impossible to quantify over everything.

But:

- either 'everything' in (5) has a restricted scope, and then (5) is false;
- or 'everything' is unrestricted, but then (5) is a counter-example against the generality-relativist's own view.

First way out (p. 428): the distinction between the language L and the metalanguage ML:

(5L) It is impossible to quantify in L over everything.

But (5L) is not able to capture generality-relativism in full generality.

Second way out (p. 430): using a single language L, but invoking different contexts of discourse:

(13) For any context C_0, there is a context C_1 such that not everything that is quantified over in C_1 is quantified over in C_0.

Still, the assertion
there is a context C_1 such that not everything that is quantified over in C_1 is quantified over in C_0
cannot be true as uttered in context C_0.

Since C_0 can be *any* context, “(13) is not true in its intended sense as uttered in any context.” (p. 430)

Conclusion: If generality-absolutism is inconsistent, generality-relativism is self-refuting.

Common mistake shared by both opponents: *everything* corresponds to something fixed,

- either a universal class that can be quantified over, but is not set-theoretic in nature
- or a set-theoretic universe that cannot be quantified over, out of which every domain is lifted, but which is not a domain itself.

Open avenue: the content of *everything* is not fixed, but indefinitely extensible.

Quine: “To be is to be the value of a variable.”

> *To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable. [. . .] The variables of quantification, ‘something’, ‘nothing’, ‘everything’, range over our whole ontology, whatever it may be; [. . .] a theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be true. (W.V.O. Quine, “On What There Is”, From a logical point of view, p. 13-14)*

- Ontology becomes the collection of all the things the existence of which a given theory is committed to.
- Ontology begins as a local feature: the ontology of a particular science (whose language has been logically regimented).
- Still all ontologies should fit into a single overall conceptual scheme:

> *Now how are we to adjudicate among rival ontologies? Certainly the answer is not provided by the semantical formula "To be is to be the value of a variable"; this formula serves rather, conversely, in testing the conformity of a given remark or doctrine to a prior ontological standard. [. . .] Our ontology is determined once we have fixed upon the over-all conceptual scheme which is to accommodate science in the broadest sense. (W.V.O. Quine, "On What There Is", p. 15-17)*

There is no “fact of the matter” about which conceptual scheme we should use to build our representation of the world.

Ontology is a matter of choice in the context of a global *dynamic adjustment*.

This is a clear example of completely relativist universalism.

Kit Fine, “Relatively Unrestricted Quantification” (2003).
Modal “expansionist” approach toward extensibility.

Each quantification $\forall y$ refers to some domain x, in such a way that x cannot be considered in the same overall domain as all the y's.
For instance, $\exists x \forall y\ (y \in x \leftrightarrow y \notin y)$ is inconsistent if quantification is taken to be absolutely unrestricted.

Russell's paradox shows that x cannot be in the same universe as all the y's: x prompts an extension of the interpretation hitherto taken to be unrestricted. The current interpretation of quantification (over y) is, so to speak, updated and replaced with an extension thereof.

The extensibility of a domain into a broader domain is presented by Fine in modal terms: an interpretation I is extensible if there *possibly* exists an interpretation extending it.

Extensibility is also conceived of as “postulational:”

> *Associated with the condition* $\forall y (y \in x)$ *will be an instruction or ‘procedural postulate’,* $!x \forall y (y \in x)$*, requiring us to introduce an object* x *whose members are the objects* y *of the given domain. [. . .] Thus* $!x \forall y (y \in x)$ *serves as a positive injunction on how the domain is to be extended rather than as a negative constraint on how it is to be restricted. (Fine, p. 37)*

Fine, cont'd:

> *[Under both the universalist and the restritionist accounts, the old and new domains are to be understood as restrictions.] Under the expansionist account, by contrast, the new domain is not to be understood as a restriction at all but as an expansion. [. . .] We might say that the new domain is understood from 'above' under the universalist and restrictionist accounts, in so far as it is understood as the restriction of a possibly broader domain, but that it is understood from 'below' under the expansionist account, in that it is understood as the expansion of a possibly narrower domain. (Fine, p. 38)*

Any domain of quantification *can* be extended, because some new object can be added to it. But this new object is not available prior to the extension itself.

In other words, an extended domain is not the restriction of some extension that would be given beforehand.

The extensibility does not derive from an anticipated extension (in comparison to which a domain appears as restricted), it is the other way around: the extension derives from the extensibility.

The modality involved in extensibility should not be thrown back upon the usual modal concepts metaphysically understood.

Shaughan Lavine, "Something About Everything" (2003). Lavine compares an open scheme such as $\phi(n) \to \phi(Sn)$ to the corresponding universally quantified sentence plus $(\forall x)(\phi(x) \to \phi(Sx))$.

> *[. . .] a universally quantified sentence only makes a claim about all the members of its universe of discourse [. . .] if it is possible that someting could have existed (that is, been a member of the [assumed to exist] unrestricted universe of discourse) that in fact does not, then the full scheme expresses commitments we would have had had such things existed that are not expressed by the universally quantified form. Such commitments might follow from the claim that the universally quantified form is not only true, but necessarily true.*
> *(Lavine, p. 122)*

Lavine, cont'd:

> *The use of necessitation in combination with unrestricted quantification poses a problem for the advocate of unrestricted quantification. The universe of discourse is supposed to be unrestricted, and so it should include all states of affairs, possible worlds, or whatever else one might think a robust notion of necessity requires. It ought therefore to be possible to express necessity without any modal apparatus that goes beyond what can be expressed using unrestricted quantification. The apparent need for a necessitation operator already casts doubt on the claim that the nominally unrestricted quantification is unrestricted in a sufficiently robust sense.*
> *(Lavine, p. 122)*

Irony: With the idea of extensibility, modal notions sneak in, whereas modal notions have been the traditional enemy of logical universalism.

(See also Quine, “Reference and modality” + “Three Grades of Modal Involvement”.)

Here is the **basic dilemma about "absolutely everything"**:

- Either the range of unrestricted quantification includes all possible states of affairs, and all the possible entities that compose them, but then it becomes impossible to consider that the universe of absolutely everything could have been different and, in particular, that it could be extended.
- Or the range of unrestricted quantification does not include all possible things, but then it does not contain absolutely everything.

Fine and Lavine both defuse the dilemma through the idea that absolute generality should correspond **to a genuine open-ended progression** where each domain is **not** anticipated as the restriction of its own extension (or as the restriction of the background universe that underlies all domains, as in set-theoretic semantics).

Absolute generality is not unrestricted generality (as if some pristine universal domain were given originally), but covers everything, whatever everything *can be* (which is not the same as "everything *possible*").

Conclusion

Ontology cannot be renounced. But it certainly cannot be naive.

It is already the case since Kant.

It is all the more the case since Russell.

But, actually, logic has always been exactly this: non-naive ontology.